

Numerical Method to Determine Effective Thermal Conductivity of Perforated Plate Matrix Heat Exchanger Surfaces and its Experimental Validation

Anish K. John, K. Krishnakumar, Rathish T. R.

Abstract—For high end heat exchanging applications in cryogenics and space application, compactness and effectiveness are the key factors which determine the performance of heat exchanger. Matrix heat exchanger with perforated plate heat surfaces has both criterions to suit into these applications. High thermal conductivity plate and low thermal conductivity spacers are stacked together to obtain low longitudinal heat conductance and thus near cent percent effectiveness to the heat exchanger. In this work, numerical methods are used to obtain the effective thermal conductivity of different types of perforated plates using ANSYS software. The effect of porosity and perforation perimeter factor, which is a ratio between perimeter of the perforation to the plate thickness, is studied. The experimental data confirms the design approach.

Index Terms— Ansys, Effective Thermal Conductivity, Matrix Heat Exchanger, Numerical Analysis, Perforation Perimeter Factor, Perforated Plates, Porosity.

1 INTRODUCTION

FOR high end heat exchanging applications in cryogenics and aerospace, compactness and near cent effectiveness are the key factors which determine the performance of heat exchanger. Matrix heat exchanger with perforated plate surfaces has both criterions to suit it best to these applications. The performance of refrigerators, liquefiers and separation units is strongly dependent on the effectiveness of the heat exchangers used. Their performance affects the sizing and design of other major equipments including compressors and their power drivers, thermodynamic considerations make cryogenic processes very sensitive to the heat exchanger performance. A drastic reduction in the production of cryogenic liquid is predicted by Atrey [1] if the heat exchanger effectiveness (ϵ) slightly reduces from the ideal. The low values of attainable coefficient of performance [2] and the resulting high cost of refrigeration make it economically sensible to use a more effective and expensive heat transfer equipment. Apart from having a high effectiveness, cryogenic heat exchangers also need to be very compact, i.e. they must accommodate a large amount of surface area in a small volume. This helps in controlling heat exchange with the surroundings by reducing exposed surface area. Besides, a small mass means a smaller cooling load and a faster cooling time for refrigerators. This is

particularly important for small refrigerators operating at very low temperature and this requirement to attain high effectiveness and high degree of compactness together in one unit led to the invention of matrix heat exchangers by Mc Mohan, Bowen and Bleyele [3] et al. in 1949.

A compact heat exchanger is generally defined as one which incorporates a heat transfer surface having a high "area density". It possesses a high ratio of heat transfer surface area to volume. Shah [4] arbitrarily defines a compact heat exchange surface as one that has an area density (β) greater than $700 \text{ m}^2 / \text{m}^3$. Perforated plate matrix heat exchangers (MHE) as shown in fig. 1, essentially consist of a stack of perforated plates made of high thermal conductivity material such as copper alternating with spacers made of low thermal conductivity material such as paper. The stack of alternate high and low thermal conductivity materials is bonded to form a monolithic block under mechanical pressure. The gaps in between the plates ensure uniform flow distribution (by continuous reheadering) and create turbulence which enhances heat transfer. Small perforations (diameter ranging from 1.5 mm to less than 0.4 mm) are made into the plates so that a large heat transfer coefficient and high surface area density (up to $6000 \text{ m}^2 / \text{m}^3$) is achieved. The ratio of the plate thickness (length of the hole in the plate) to the diameter of the hole is on the order of 0.75, therefore, the thermal and hydrodynamic boundary layers do not become fully developed within the perforations, which results in high heat transfer coefficients and correspondingly high friction factors. The flow within the small holes in the perforated plate is generally laminar. Due to their small hydraulic diameter and the low density of gases,

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the surfaces are usually operated in the Reynolds number range $500 < Re < 1500$ [5]. The correlation for the heat transfer coefficient within the small perforations for laminar flow within tubes is by provided Hubbell and Cain in 1988 [6]. The spacers, being of low thermal conductivity material, also help in reducing axial conduction and consequent deterioration of performance. The spacers perform multiple roles such as reducing the longitudinal heat conduction through the walls, reducing the flow maldistribution by reheadering the flow in each spacer and interrupting the boundary layer, thus enhancing the heat transfer coefficients.

NOMENCLATURE

A:	Area of plate, m^2
c:	Specific heat at constant pressure of the fluid, J/kgK
d:	Perforation diameter of the plate, m
k:	Thermal conductivity of the matrix material, W/mK
l:	Plate thickness, m
n:	Number of perforations
P_r :	Perforation perimeter factor
p:	Porosity
Q:	Heat transferred, W
\dot{q} :	Heat generated, W
T:	Temperature, K
t:	Time, s
x:	Distance between temperature measuring points, m
β :	Area density, m^2/m^3
Subscripts	
eff:	Effective
p:	Plate

2 LITERATURE REVIEW

Several types of compact cryogenic heat exchanger have been reviewed by Dilevskaya [7] and Zablotskaya [8]. A detailed review regarding the heat exchanger models in cryogenic applications are provided by Pacio and Dorao [9] and Venkatarathnam and Sarangi [10]. Venkatarathnam and Sarangi trace the chronological development of the MHE and different methods of fabrication, heat transfer and fluid flow characteristics and design and simulation procedures. A systematic review of the state of art and challenges in modelling cryogenic heat exchangers is presented by Pacio and Dorao. Krishnakumar, Anish and Venkatarathnam [11] discussed various transient test techniques that are used for the determination of heat transfer characteristics of high NTU heat exchanger surfaces. Reay [12], in his paper, briefly reviewed the benefits of compact heat exchangers, as well as their limitations and provided description of a number of types of compact heat exchanger. Li, Flamant, Yuan, Pierre Neveu, Lingai Luo [13] gave a review on performances of compact heat exchangers (CHEs),

the structures of the CHEs and their heat transfer enhancement mechanisms. Then, different heat transfer enhancement technologies in CHEs are compared and their thermo-hydraulic performances are analyzed on the basis of available correlations for heat transfer and friction factor developed by various investigators quoted in the open literature.

The cylindrical holes in the plates may be arranged in square or hexagonal array. Published data show that for porosity up to 0.6, which mostly covers the MHE applications, the K_{eff} values for square and hexagonal arrays are very close. Thus, the K_{eff} values for square array can also be used for hexagonal array (up to porosity 0.6). S. Sunil Kumar and T. K. Nandi [14] presents a numerical model using finite difference method for predicting k_{eff} of plates having holes in square array. The results obtained are compared with the available experimental data and with the data obtained from analytical expressions. An appropriate correlation suitable for the design of MHEs is also identified.

White et al. [15] presents a numerical model of a perforated plate heat exchanger that accounts for axial conduction, external parasitic heat loads, variable fluid and material properties, and conduction to and from the ends of the heat exchanger. The numerical model is validated by experimentally testing several perforated plate heat exchangers that are fabricated using microelectromechanical systems based manufacturing methods. The experimental testing demonstrates the ability of the numerical model to accurately predict both the overall performance and the internal temperature distribution of perforated plate heat exchangers over a range of geometry and operating conditions. The parameters that were varied include the axial length, temperature range, mass flow rate and working fluid.

Cajas et al. studied [16] the effect of a circular perforation on the heat conduction in a plate with variable thermal conductivity. The corresponding heat equation is written in non-dimensional form, depending on three non-dimensional parameters: the aspect ratio of the plate; the radius of perforation related to the width of the plate and the non-dimensional thermal conductivity parameter, which measures the variation of the thermal conductivity with temperature. This equation is solved by two different approaches. The first one is numerical, using a coordinate fitted grid generator, and the second one is analytical, by developing an asymptotic solution for small values of the radius of perforation related to the width of the plate and the non-dimensional thermal conductivity parameter assumed to be very small compared with unity. The non-dimensional heat flux given by the Nusselt number has been obtained and the numerical results have been compared with the analytical solution. Excellent agreement is achieved even for values of the radius of perforation related to the width of the plate up to values close to 0.4.

Hayes et al [17] investigated heat transfer and fluid flow

characteristics through a porous medium using numerical simulations and experiment. For the numerical simulations two models were created: a two-dimensional numerical model and a Fluent™ computational fluid dynamics (CFD) porous media model. The results of the numerical models were compared to the experimental data in order to determine the accuracy of the models. Numerical simulations have been carried out to investigate the turbulent heat transfer enhancement in the pipe filled with porous media by Yang & Hwang [18]. Two-dimensional axisymmetric numerical simulations using the $k-\epsilon$ turbulent model is used to calculate the fluid flow and heat transfer characteristics in a pipe filled with porous media. The numerical results show that the flow field can be adjusted and the thickness of boundary layer can be decreased by the inserted porous medium so that the heat transfer can be enhanced in the pipe. Fluid flow and heat transfer characteristics in a channel with staggered porous blocks were numerically studied by Li et al. [19]. The Navier–Stokes and Brinkman–Forchheimer equations were used to model the fluid flow in the open and porous regions, respectively. Coupling of the pressure and velocity fields was resolved using the SIMPLER algorithm. The effect of Darcy number, Reynolds number, porous block height and width on the velocity field were studied. In addition, the effects of the above parameters as well as the thermal conductivity ratio between the porous blocks and the fluid on the local heat transfer were analyzed. Hou Yu et al. [20] presented a numerical model for the design and thermal analysis of woven wire screen matrix heat exchanger (WSMHE) for cryogenic applications. The influence of wall thermal resistance, axial conduction, parasitic heat load and properties variation was taken into account, which is neglected in the traditional effectiveness–NTU method but important for compact cryogenic heat exchangers. The proposed numerical method is verified by effectiveness–NTU method under specific conditions, then it is tested according to experiment data, and a well agreement was obtained.

Numerical solution of the heat exchange equations and methods are given for determining the characteristics of the perforated plate heat exchanger required for a given application, the properties of the perforated plate matrix material and the properties of the working fluid are also provided by Nilles et al. [21]. Experimental data confirm the design approach which addressed pressure drop, plate conduction and longitudinal thermal conductivity in the exchanger and entrance effects.

3 NUMERICAL SIMULATION

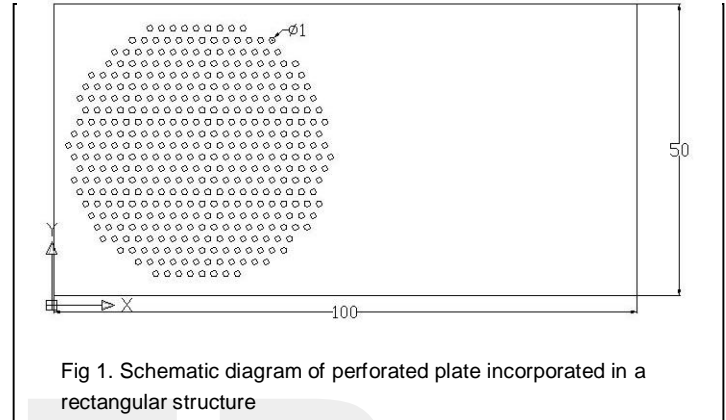
In this work, the numerical simulation details are explained for obtaining the effective thermal conductivity of perforated plate matrix heat exchanger surfaces. Each plate is modelled using numerical model based on finite element technique. The geometry was created using the software Pro-Engineer™ and

analysed using ANSYS 14™.

3.1 Assumptions Made for Investigation

- Steady state thermal analysis is carried out to determine the nodal temperature.
- Radiation heat transfer effects inside the perforated plate can be neglected.
- The physical properties of the perforated plate remain constant throughout the temperature range.

3.2 Geometry & Mesh



Model under study is a perforated plate having diameter 50 mm and perforation diameter of 1 mm. The 3D model of circular perforated plate is then incorporated in a rectangular structure having dimension of 50 x 100 mm² which was created using Pro-Engineer and imported to ANSYS. The rectangular section is provided for transferring uniform heat along the perforated plate. The physical model under study is shown in figure 1. The element used for modelling is SOLID 70, which is a 3D brick shaped structure with eight nodes. It is having temperature as its degree of freedom at each node. The isotropic properties such as thermal conductivity, density of the material and heat capacity are provided as that of copper.

The loads applied on the model are constant heat flux, constant temperatures and convections. Convections are surface loads applied on exterior surfaces of the model to account for heat lost to surrounding fluid medium. Heat fluxes are also surface loads acting on the inlet surface. A positive value of heat flux indicates heat flowing into the element. Constant temperatures are DOF constraints usually specified at model boundaries to impose a known, fixed temperature. Constant heat flux condition of 257333 W/m² and constant temperature condition of 30°C were defined for two end faces of this model. Convection condition was defined for the perforation surfaces. All other sides are considered to be adiabatic surfaces. Meshing was done using free tetragonal grids. Mesh smart size level 1 having scaling factor 0.2 was used to mesh the volume and then refined the mesh over the elements and a meshed perforation is shown in fig 2.

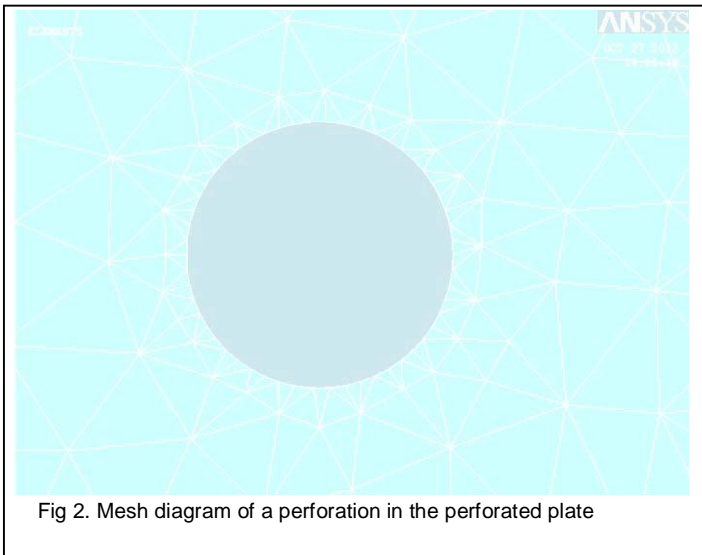


Fig 2. Mesh diagram of a perforation in the perforated plate

3.3 Governing Equation

Steady state thermal analysis of the perforated plates was done in ANSYS 14. In steady state thermal analysis, the heat balanced equation obtained by applying conservation of energy is the basis. This equation is combined with the Fourier law equation to obtain a modified heat balance equation. This equation is integrated over the element volume. After defining the boundary condition the model were solved using sparse direct solver. Sparse direct solver solves by direct elimination rather than iterative solutions. In this, the factorization of an initial very sparse linear system of equations into a lower triangular matrix followed by forward and backward substitution using this triangular system is done. The post processing was also done. The post processing involved the nodal solution plot. The nodal temperatures were obtained and graphed. General equation to be solved is:

$$\rho c \left(\frac{\partial T}{\partial t} + V \cdot \nabla T \right) = \nabla \cdot (K \nabla T) + \ddot{q} \quad (1)$$

3.4 Fluent Methodology

At the inlet section, heat flux was considered as input parameter. For the outlet section, input was temperature condition. It was solved numerically for plain plate and plates with different perforations. The steady state temperature at the initial and final position near to the plate perforations, which is at a distance of 50mm away from the inlet section, was noted. Using the Fourier equation, the effective thermal conductivity was obtained. This process was repeated for different types of perforation plates.

For comparing the numerical predictions with the experimental results, effective thermal conductivity can be experimentally determined using the Fourier equation:

$$Q = -k \cdot A \cdot \frac{dT}{dx} \quad (2)$$

Porosity, p is given by [21]

$$p = n \cdot \frac{d^2}{d_p^2} \quad (3)$$

Effective thermal conductivity is given by

$$k_{eff} = (1 - 1.16 p) k_p \quad (4)$$

4 EXPERIMENTAL VERIFICATION

Experimental investigations were carried out on perforated plate to find out the effective thermal conductivity as shown in fig 3. The circular test specimen was placed in between two rectangular extensions with semicircular ends of same radius. Care was taken to avoid any air resistance between the contact surfaces. A heater was provided on the first rectangular section, which was connected to a variable power source. Calibrated multimeter was used to measure the load from the autotransformer. T type thermocouples of size 32 gage were used to measure the temperature at two ends of perforated plate along the diameter in the longitudinal direction. Asbestos insulation is provided to insulate heat flow to the atmosphere. A data acquisition system, Agilent 34970A, was used to record the temperature from different points. The test procedure consists of providing heat along the diameter of the perforated plate. The steady state temperature difference is noted from two specified points. The effective thermal conductivity is obtained from the temperature difference between the perforated plate ends. In the present work, thermal conductivity of a plain plate is compared with that of the perforated plate experimentally to obtain effective thermal conductivity ratio, which is a ratio of thermal conductivity of plain plate to thermal conductivity of perforated plate. Experiments were repeated for perforated plates with different porosities and perforation perimeter factor, P_f , which is the ratio of perimeter of the perforation to plate thickness. The perforated plates are shown in figure 4. Based on the uncertainties in the measurements, the uncertainty in the estimation of k was 10%. This was calculated by the Kline and McClintock method.

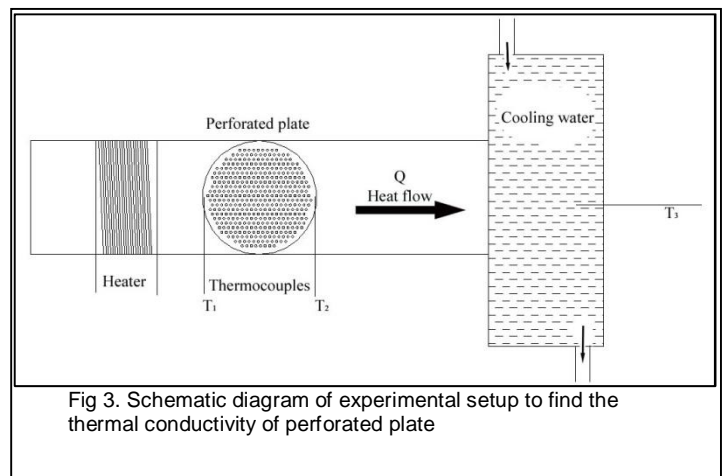


Fig 3. Schematic diagram of experimental setup to find the thermal conductivity of perforated plate

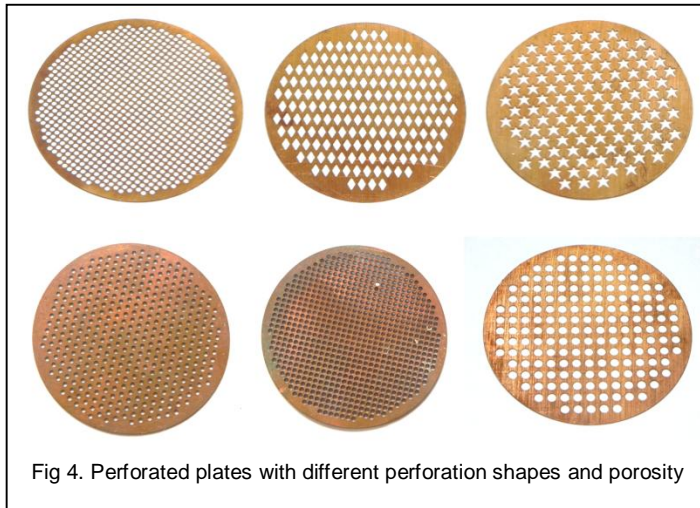


Fig 4. Perforated plates with different perforation shapes and porosity

5 RESULTS AND DISCUSSION

5.1 Variation of Porosity with Effective Thermal Conductivity Ratio

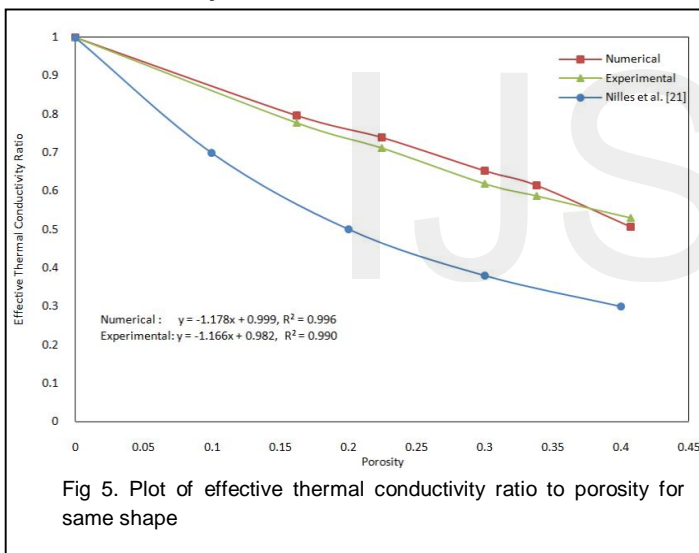


Fig 5. Plot of effective thermal conductivity ratio to porosity for same shape

A simple expression for effective thermal conductivity for a random hole distribution is given as,

$$k_{eff} = (1 - p)k_p \tag{5}$$

Nilles, Clakins, Dingus and Hendricks [21] provides a better fit of within 5%, is obtained using the form,

$$k_{eff} = (1 - 1.16p)k_p \text{ for } p < 0.68 \tag{6}$$

Figure 5 is a plot of the effective thermal conductivity ratio with porosity. For our work, the perforations provided are circular and the equation obtained is

$$k_{eff} = (0.999 - 1.178p)k_p \tag{7}$$

with the R squared static indicates that the model as fitted explains 99.6 % of the variability in the dependent parameters of k_{eff}/k_p and p respectively as shown in figure 5. The linear trendline from the numerical values obtained from this work, shows similarity with that obtained from Nilles et al. The de-

viation observed in the values of effective thermal conductivity may be due to different l/d ratio. There is no similar work for perforated plate matrix heat exchanger available in open literature.

5.2 Effect of Perforation Perimeter Factor with Effective Thermal Conductivity Ratio

Different shapes like circular, rhomboid and star are evaluated at constant porosity of 0.16 and 0.3. The shape of the perforation is quantified by a term of perforation perimeter factor, P_f , which is a ratio between the perimeter of the perforation to the plate thickness. Perimeter is found out by measuring the physical edge length which is in contact with the fluid flow, and not by the mean hydraulic diameter. The perforation perimeter factor for circular, rhomboid and star shape is found out to be 10.5, 13.3 and 24 respectively. The k_{eff}/k_p value is found to be in the increasing order as shown in figure 6 for star, rhombus and circular shaped holes same as the increasing value of perforation perimeter factor. Star shaped perforation is having high effective thermal conductivity ratio, k_{eff}/k_p may be due to the fact that fluid interaction with plate surface is more and also due to enhanced turbulence when passing through star shaped perforation. As the perforation inner perimeter increases, the area available for heat transfer within the perforation is more and hence a higher value for k_{eff}/k_p . The inner perforation perimeter is maximum for the star shaped perforation and least for circular for same porosity.

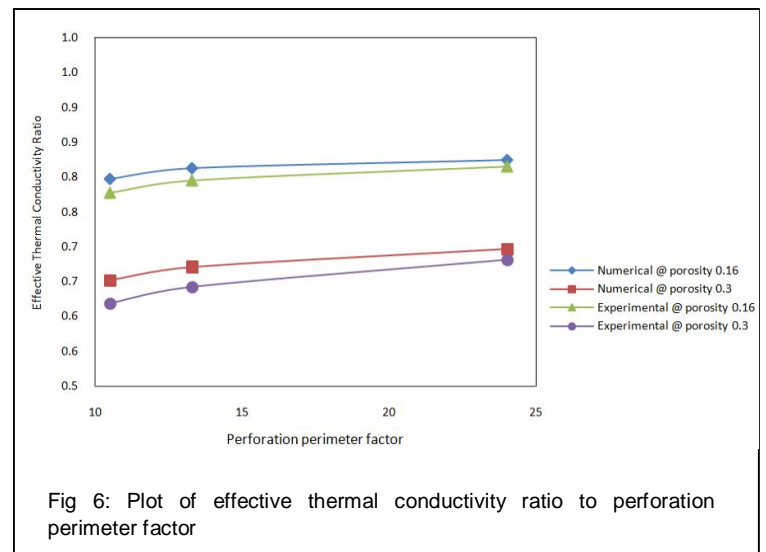


Fig 6: Plot of effective thermal conductivity ratio to perforation perimeter factor

6 CONCLUSION

The conclusion from this paper is as follows:

1. The effect of porosity on effective thermal conductivity ratio for perforated plate is numerically determined and experimentally validated.
2. The effect of shape of perforation on effective thermal conductivity ratio for perforated plate is numerically determined and experimentally validated.
3. A new parameter perforation perimeter factor, P_f is coined to quantitatively represent the shape of the perforation.

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